Quantum Error Correction for Computing and Networking

Doctoral thesis booklet

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Introduction and motivation

The thesis focuses on two areas of error correction and hence is divided into two main parts. The first part deals with traditional quantum error-correcting codes tailored for noisy quantum computers, while the second part concentrates on the distillation of noisy states used in quantum networks. The thesis integrates findings from three independent studies which collectively aim to progress in the fields of quantum error correction, quantum memory and multipartite entanglement distillation. They emphasize the significance of optimized protocols for preserving and distilling quantum states in practical quantum computing applications.

In the first part of this thesis, we focus on traditional quantum error correction codes and quantum memories. Quantum memory, storing qubits for a sufficiently long time, is a key ingredient of almost any application in quantum communication, computing, and sensing. Designing long-lived quantum memories is one of the recent challenges in the field of quantum technology. It can be achieved by improving the lifetime of each physical qubit and storing the logical quantum information in a collective quantum state of multiple physical qubits. The theory of quantum error correction proves that quantum codes are promising platforms for the fault-tolerant storage of quantum information. Typical quantum memory consists of the following processes: encoding the quantum information into the quantum code, idling time, correction, and recalling the quantum state [1]. (See Fig. 1.) Recalling is the inverse process of encoding when the logical information is refocused onto one physical qubit.

A large zoo of quantum codes is applicable for quantum memory [2]. One of the most promising platforms is the surface code because, in theory, the logical errors could be reduced by scaling up the size of the system if the physical errors are below a threshold. Moreover, it can be realized on a 2D square grid of physical qubits with connectivity only between the neighbors. However, because of initialization, gate, and measurement errors the *break-even point* - i.e., preserving an arbitrary quantum state with surface code longer than a single idle qubit's lifetime - was reached in 2024 [3].

The simplicity of the repetition code comes from the fact that it requires only linear connectivity between the qubits, and for the implementation, only three qubits are enough [4]. Furthermore, repetition code is the only error-correcting code that was realized on semiconductor qubits. Phase-flip code-based quantum memory without mid-circuit measurements was implemented in Ge-based and in Si-based spin-qubit devices, but due to the imperfection of the gates, the lifetime of the logical qubit was smaller than the lifetime of an idle physical qubit [5].

In the first part of this thesis, we aim to analytically determine the parameter regime (gate error, relaxation T_1, and dephasing T_2^*), where the repetition code-based quantum memory beats the break-even point.

In the second part of the thesis, we focus on developing entanglement distillation protocols. Quantum entanglement provides an advantage to quantum systems over classical ones for various applications in the fields of communication, computation, and sensing. The multipartite-entangled states are widely studied in the literature because their quantification, characterization, and manipula-

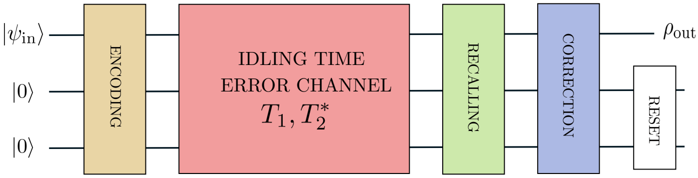

Figure 1: The schematic draw of a quantum code-based memory. In the decoding step, the initial state Ψ_{in} is entangled with a few ancillary qubits initialized in the ground state. During the idling time, every qubit is affected by the noise sources, e.g., dephasing and relaxation. After the correction, the state is recalled to one of the physical qubits. A reset of the ancilla qubits is necessary if we want to repeat this process. In many applications, the order of the recalling and correction processes is reversed.

tion are richer than in the bipartite case. Moreover, multipartite entanglement is a potential asset in several applications, such as conference key agreement, measurement-based quantum computing, quantum codes, and quantum metrology just to name a few. The Greenberger-Horne-Zeilinger (GHZ) state is a maximally entangled multipartite state that has been already realized at different platforms, including photons, trapped ions, semiconducting qubits, and superconducting qubits. Because of the non-perfect generation, before the application, it may be necessary to distill these states.

Entanglement distillation[6] is a transformation of some less entangled state into a smaller number of more entangled states, using only local operation and classical communication (LOCC). Distillation of multipartite entangled states and especially, GHZ states has attracted considerable attention [7, 8]. Two types of

GHZ state distillation protocols were studied in the literature, the asymptotic and the one-way protocols. In the case of the former, during one step, one or more copies of a distorted GHZ state are sacrificed to increase the fidelity of one state to the pure GHZ state. Repeating the steps (with the same operations) one after the other produces asymptotically a perfect GHZ state. The downside of this protocol is that the number of necessary states increases rapidly (exponentially) with the number of iterations. The one-way protocols, such as hashing or breeding protocols operate on an asymptotic large ensemble of entangled states.

We investigate the distillation capability of an asymptotic LOCC scheme on tripartite GHZ states. By numerically and analytically approaches we derive efficient protocols to asymptotically prepare high-fidelity GHZ state from moderately but arbitrarily distorted GHZ states. It means, that our protocols correct both the coherent and incoherent errors of the input states. Our protocols have further advantages: (i) converge subexponentially to the pure GHZ state as a function of the number of iterations, (ii) the unitary operations of the protocols are relatively simple and easy to gate decompose.

Thesis points

1 Break-Even Point in Quantum Repetition Code[9]

The use of quantum code-based memories is explored to enhance the lifetime of qubits and exceed the break-even point, which is critical for the implementation of fault-tolerant quantum computing.

- It was theoretically demonstrated that the phase-flip repetition code can preserve arbitrary quantum information longer than the lifetime of a single idle qubit in a dephasing-time-limited system

- Analytical calculations revealed the conditions for achieving optimal performance, including idling time thresholds, gate error probabilities, and the number of error correction cycles.

- It was shown that resetting ancillary qubits and optimizing the error correction cycles according to gate error rates significantly improves the fidelity of preserved quantum states.

These results provide practical guidelines for implementing efficient quantum memories, particularly in semiconductor quantum devices.

2 Training Quantum Circuits for GHZ State Distillation[10]

A quantum protocol was presented, involving unitary transformations acting on two qubits and subsequent measurements and post-selection, optimized for the purpose of distilling GHZ states in an LOCC scheme.

- By optimizing a quantum circuit, employing a variational optimization algorithm, it was demonstrated that integrating double iteration of the circuit into the cost fidelity function yields effective distillation results.

- The even-odd oscillatory pattern of iterations was identified, with fidelity improving significantly when restricted to even iterations.

- The protocol was shown to be hardware-efficient and applicable to noisy GHZ states, making it relevant for recent quantum architecture prototypes.

3 Practical Distillation Protocols for GHZ States

A double-iteration protocol was systematically developed, providing a mathematical framework for the transformation processes involved and emphasizing the role of unitary operations in correcting arbitrary small errors in the initial states.

- It was demonstrated the protocol achieves subexponential convergence toward high-fidelity GHZ states, tolerating moderate measurement and gate errors.

- Two types of iterative protocols were compared: the alternating protocol allows simpler gate decomposition, while the uniform protocol accommodates larger coherent distortions.

- Our protocol effectively corrects small arbitrary distortions in GHZ states while maintaining operational simplicity, making it suitable for practical quantum computing applications.

The publications providing the basis for the thesis points

- Áron Rozgonyi and G. Széchenyi, “Break-even point of the phase-flip error correcting code”, New Journal of Physics 25, 10.1088/1367-2630/acfba5 (2023).

- Áron Rozgonyi, G. Széchenyi, O. Kálmán, and T. Kiss, “Training iterated protocols for distillation of ghz states with variational quantum algorithms”, Physics Letters A 499, 129349 (2024).

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