MICROWAVE AND OPTICAL MIXING WITH ATOMS

Thesis booklet

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2024

Introduction

The hyperfine transitions of alkaline atoms have played a crucial role in shaping the landscape of quantum physics since the early twentieth century. These transitions exhibit a diverse range of applications in the field of quantum technology. One notable application involves the definition of the time standard relying on the unperturbed ground-state hyperfine transition of the of the ^{133}Cs atom [1]. Moreover, the phenomenon of quantum interference between optical transitions, where hyperfine sublevels are optically coupled to an excited state revealed new physics, leading to various advancements. These include, among others, the cooling neutral atoms [2–4], reaching subrecoil temperatures without significant loss of atoms, the realization of atom interferometers [5], and the production of slow light [6], where photons can be stored in the excitation of a medium [7, 8]. The field of superconductivity has introduced an alternative approach to study the interaction between light and matter, known as circuit quantum electrodynamics (QED) [9]. This approach allows for the realization of superconducting artificial atoms with hyperfine transitions coupled to microwave fields [10]. Consequently, systems involving atoms with hyperfine structure interfacing with both microwave and optical fields have become a central focus in quantum technology. Despite significant progress, there remain unexplored territories in this domain. The intention behind the research presented in this thesis booklet was to delve into a portion of this uncharted territory, with a particular focus on systems that involves atoms with hyperfine transitions.

Short description

1. Microwave-to-optical transduction

A quantum network consists of a set of quantum processing and storage nodes distributed at different locations, interconnected by optical fibers for the transmission of quantum information through photons. Various platforms with experimentally verified relevant quantum capabilities have been proposed for the implementation of the nodes. These include nuclear magnetic resonance systems [11], single trapped ions [12], neutral atoms in optical lattices [13], single atoms in optical cavities [14], quantum dots [15, 16], color centers in crystals [17], and perhaps the most promising, superconducting circuits [18, 19]. The latter platform operates in the microwave regime, necessitating the development of coherent microwave-to-optical transducers for connecting distant superconducting quantum circuits via optical fibers. Ensembles of atoms are a viable candidate for this, offering good coherence properties and strong dipole transitions for efficient coupling to optical photons [20–22]. The concept of trapping atoms on surfaces using nano-fabricated wires, introduced by Schmiedmayer [23], resulted in the implementation of an Atom Chip-based converter, wherein the hyperfine transitions of cold trapped atoms are coupled to a superconducting microwave resonator [24–26]. In many situations of interest, such atoms can be modeled as three-level systems with a Λ configuration of levels. A single microwave photon [27, 28] can be converted into a spin-wave excitation of the atomic hyperfine sublevels, which, in turn, can be transferred to a single optical photon in a stimulated Raman process that is inherently reversible [20, 21]. In the Λ configuration, a hyperfine transition of a single atom does not have sufficient interaction strength with a microwave photon to make a practically useful transducer. The natural mitigation is to use large ensembles of atoms [26].

During my PhD studies, one aspect of the research explored the emission of photons from a coherently prepared atomic ensemble. Since the generated optical photons are envisaged for applications in quantum communication, we examined how efficiently such photons can be coupled into guided Gaussian modes focused on an optical fiber by a paraxial optical array. We focused on the spatial profile of the generated radiation coupled into the fiber, which is encoded in the spatial overlap integrals of the scattered field with the read-out Gaussian mode. We considered an ensemble of atoms in a cylindrically symmetric harmonic trap and examined the spatial overlap integrals through both analytical and numerical approaches. Since for a given direction of the optical fiber the guided Gaussian modes form a broadband one-dimensional continuum around the frequency of the emitted radiation, we treated the radiated intensity within the Born-Markov approximation, similar to free-space spontaneous emission. Our research aimed to address the question of the optimal geometrical conditions for enhancing the collection efficiency of the generated optical photons. This optimization involved considering the incoupled photon rate across a parameter space that included the number of atoms, the trap geometry, the beam waist of the Gaussian mode, and the orientation of the incoming driving field relative to the direction of the optical fiber.

We examined once more the microwave-optical conversion process this time occurring in a degenerate quantum gas—Bose-Einstein condensate (BEC)—with large coherence length compared to the optical wavelength. Already in the early days of BEC experiments, two essential features of light scattering on a condensate was noted: superradiance due to bosonic enhancement, and the possibility for the light to excite density waves that extend coherently over the whole atomic sample [29, 30]. Recognizing that light scattering can create excitations in the BEC that are associated with atomic momentum, in addition to the internal atomic state, we incorporated the atomic external, motional degree of freedom into our analysis to describe momentum transfer in the photon recoil. In particular, the BEC can take away almost arbitrary momentum from the photonic part of the process without significantly altering its energetics. This is attributed to the extremely flat dispersion relation of BEC excitations on the momentum scale relevant to optical photons, owing to the large atomic mass. We used a second-quantized description in which the equations of motion can be written straightforwardly in the single-excitation subspace. The amplitudes describing the generated optical radiation from the atomic source were used to calculate and discuss the intensity of the coupled photons into the optical fiber.

2. Photon-blockade breakdown

A nonlinear atomic media placed within an optical resonator opens the way to explore the nonlinear aspects of light-matter interactions. An illustrative example of this nonlinearity is the phenomenon of blockade breakdown. In the low drive intensity regime, the cavity suppresses transmission. However, at a critical point or within a specific domain of the drive intensity, the blockade abruptly breaks down, the cavity allows transmission. In certain cases, the critical domain displays bimodality, indicating that the cavity filled with atoms has two stable stationary solutions. These solutions correspond to states with different cavity photon numbers—one characterized by a low photon number (dim state) and the other with a high photon number (bright state). The realization of bistability within a driven-dissipative atom-cavity system can take on different forms. One example for this nonlinear nature of light-matter interaction is optical bistbility (OB) [31–34]. It was initially observed in saturable resonators [35] and semiconductors [36]. In principle, however, a few atoms or even only a single atom could exhibit the characteristics of optical bistability [37, 38]. The nonlinear input–output relation underlying this effect can be adequately described by the steadystate solutions of the Maxwell–Bloch equations [39], commonly referred as the semiclassical model.

Much more recently, the photon-blockade breakdown (PBB), another phenomenology that involves intensity bistability but occurs in the "extremely strong" coupling regime of an interacting atom-mode system described by the driven–dissipative Jaynes–Cummings model has been proposed [40–43] and experimentally observed on the circuit QED platform [44–46]. An intuitive picture of the PBB phenomenology for a coupled system of a single two-level atom (qubit) and a single mode goes as follows: a drive tuned close to resonance with the bare frequency of the qubit-mode subsystems cannot excite the system prepared in the ground state [47–50], due to the enhanced Rabi splitting caused by the strong qubit-mode interaction. However, for any such drive detuning, there exists a region in one of the Rabi subladders where the ladder spacing matches this detuning. Such a region can accommodate a bright state, that can be reached by a combination of multi-photon transitions [41, 51, 52] and photon-number increasing quantum jumps if the drive is strong enough—the breakdown of photon blockade. On the theoretical front, in contrast to OB with its semiclassical backdrop, PBB was shown to be related to the neoclassical Jaynes–Cummings theory [42]. The boundaries of the domain of bistability in the parameter space and the strong-coupling thermodynamic limit are correctly predicted by the neoclassical theory. While the semiclassical model has been widely used in the study of light-matter interaction, neoclassical theory has remained somewhat of an undercurrent for decades. The neoclassical theory, applicable in the limit of zero atomic decay, assumes a pure qubit state, which is equivalent to the pseudospin of unit length. This entails that the qubit cannot be entangled with the mode, which is at least surprising for an interacting quantum system. Although the neoclassical theory has found indirect justification in the PBB scenario in the shape of the phase diagram and the nature of the thermodynamic limit, its fundamental assumption [42, 53]— pseudospin with unit length— has not been directly tackled. It is highly nontrivial what a real interacting and driven-dissipative quantum system would do in the zero or, even more interestingly, the small qubit decay case.

We directly studied in the language of quantum-jump Monte Carlo trajectories generated with the C++QED framework [54–56] for the strongly interacting driven-dissipative Jaynes-Cummings theory that the neoclassical model is the correct classical backdrop of the PBB phenomenology. Two values for the qubit decay rate: (i) $\gamma = 0$, and (ii) $\gamma = 0.01\kappa$ (κ is the damping rate of the cavity mode), in the strong coupling limit, $g = 100\kappa$, were considered, the second choice being made to move slightly away from the key condition of the neoclassical theory. The driving field amplitude and detuning were varied over a wide range. The neoclassical predictions were compared with the outcomes obtained from the quantum trajectory simulations: the cavity photon number, the boundaries of the bistability domain, and the length of the pseudospin. The final significant aspect of our work concerning the PBB effect involved modeling the high-lying bright state on the Rabi subladder as a pure state, precisely a coherent superposition of the Jaynes-Cummings dressed states. We compared the entanglement measures for various parameters predicted by the effective model with those obtained from the fully-quantum treatment of the strongly interacting driven-dissipative Jaynes-Cummings theory.

3. Analysis of cavity QED experimental results: models and methods

During my PhD research, I had the opportunity to actively participate in two experimental projects within our group, providing me with valuable insights into certain aspects of experimental cavity QED.

Transmission blockade breakdown transition

We explored a third type of blockade breakdown effect, distinct from the OB and PBB, and observed the time-resolved transition associated to this phenomenon. This breakdown effect involves multilevel atoms, and the transition between the two extremes of cavity transmission can be observed in the time evolution of the system. A scenario with a large number of atoms exhibiting strong collective coupling to a mode of an optical cavity was considered. Importantly, we operated in the dispersive regime, where the detuning between the atomic transition and the cavity light field was significantly larger than the strength of the single atom-mode interaction. The transition manifested between two distinct stationary states of the system, where the internal electronic state of the atoms was a pure quantum state. These states corresponded to two atomic hyperfine ground states, which were macroscopically reflected in the cavity transmission: suppressed and maximum transmission. The cavity photon number served as an appropriate quantity for tracking the time-resolved transition between these two states. Initially, all atoms were prepared in a state coupled to the mode, and due to the large collective shift, the cavity transmission was highly suppressed but not fully blockaded. Some light could infiltrate into the cavity, leading to a small atomic excitation into an intermediate state, from where the atoms could decay into a dark state decoupled from the cavity mode. The depletion of the initial state reduced the number of atoms coupled to the mode, consequently decreasing the collective mode shift. This allowed more light to enter the cavity, creating a positive feedback loop that broke down the transmission blockade.

To effectively capture certain aspects of the dynamics of the transmission blockade breakdown, we employed a simple semiclassical model. We modified the Maxwell-Bloch equations, where the usual atom-cavity interaction was complemented by an additional loss process, describing the escape to the dark states via spontaneous emission from the intermediate state. By integrating the modified Maxwell-Bloch equations, we obtained the time evolution of the intracavity photon number. The parameter values were taken from the experiment, with the escape rate being the single fitting parameter in modeling the breakdown transition.

Measuring photon statistics with single-photon counter

In cavity QED experiments, possessing a toolkit capable of distinguishing various types of light sources from a statistical perspective is crucial. Traditionally, the key function employed for this purpose is the second-order correlation function $g^{(2)}$. The diverse array of detectors designed for sensing individual photons allows for the extraction of statistical properties from a light beam by analyzing the outputs of a single-photon detector (SPD). However, in some cases, the correlation function from the measured counting sequences is not directly accessible. What can be directly obtained is the time between recorded consecutive photon pairs, known as the photoelectron waiting time. The relationship between the waiting-time distribution (WTD) and $g^{(2)}$ is well-established [57, 58] and widely used [59, 60] in the statistical characterization of a measured light source.

We employed a method to extract second-order temporal coherence from the recorded counting sequence of the detected photons. In our experimental setup, we utilized a pair of superconducting nanowire SPDs. The use of two detection channels aimed to mitigate the impact of the dead time of the detectors. From the signals of the photoelectrons, a time controller generated a sequence of timestamps, and the histogram of these timestamps led to the

WTD. According to [57], the second-order correlation function can be expressed as a self-convolution power series of the WTD. Consequently, having a time resolution shorter than the coherence time of the light source, we were able to extract information about the statistical properties of the scattered photons in our experiments.

Results in the form of theses

1. We investigated the optimal collection of photons into an optical fiber using paraxial optical elements. The collected radiation is generated through stimulated Raman scattering on Λ atoms from a microwave excitation in the ground-state hyperfine manifold. Analyzing the spatial overlap integral of the scattered field with the readout Gaussian mode, we discovered that the emitted photon is best mode-matched by the Gaussian mode with a waist equal to the width of the atomic ensemble. By choosing this optimum value for the beam waist, we demonstrated that the highest collection efficiency is achieved with a compact, i.e., narrow and short, atomic ensemble.

2. We explored the generation of optical photons in a degenerate quantum gas, considering the atomic external, motional degree of freedom to describe momentum transfer to the condensate in the photon recoil. Our findings highlighted the presence of two distinct channels for the generated radiation. One channel exhibited an intensity proportional to the number of atoms in the condensate and corresponded to isotropic radiation. This behavior arose because the condensate could absorb almost arbitrary momentum from the photonic part of the process, allowing photon scattering to occur in directions other than the phase-matched one determined by the driving field. In the other channel, where there was no momentum transfer to the condensate, superradiant behavior was observed. This channel was associated with photon scattering predominantly in the forward direction.

3. The neoclassical model as the correct theory of the PBB phenomenology was examined. Through quantum-trajectory simulations, we demonstrated that the strong assumption of the neoclassical theory—the qubit is in a pure state—aligns with the results of the fully-quantum model for the detunings $\Delta \lesssim g/4$. This alignment holds true even

in the presence of a small qubit decay, allowing the extension of the neoclassical model to cases with non-zero γ. Thus, we concluded that in the strong-coupling regime of the driven-dissipative Jaynes-Cumings model the qubit–mode interaction does not generate entanglement over quantum trajectories within a significant parameter domain.

4. With the intuitive picture of the PBB phenomenon in mind and recognizing that the anharmonicity of the spectrum of the Jaynes-Cummings model decreases for higher parts of the Rabi subladders, we modeled the bright state as a coherent superposition of the Jaynes-Cummings dressed states. By introducing this effective model, we could explain the observed trends in entanglement. Our model reproduced how entanglement emerges in the qubit-mode system with a decrease in the bright state photon number, indicating the transition of the bright state from the harmonic, higher-lying to the anharmonic, lower-lying part of the spectrum.

5. We demonstrated how a simple semiclassical model can effectively capture the dynamical breakdown of a multilevel atom-cavity blockade effect over time. This was achieved by reducing the multilevel structure into a two-level representation and introducing an additional decay channel into the Maxwell-Bloch equations of N two-level atoms. This extra escape rate causes atoms to escape from the two-level manifold and enter the dark state, not coupled to the cavity mode. As this loss process breaks down the transmission blockade, we were able to reproduce the time-resolved transition curve of the intracavity photons observed in the experiment.

6. We integrated the second-order correlation function measuring method into our measurement protocol by utilizing the directly accessible WTD of the recorded photons. After the single-photon detection of the target light, we extracted the distribution of waiting times from the measured sequence of photoelectrons' timestamps, with a time resolution shorter than the coherence time of the light source. We then calculated the second-order correlation function at different orders of the self-convolution power series of the WTD using our data post-processing algorithm.

List of publications

- Á. Kurkó, P. Domokos, A. Vukics, T. Bækkegaard, N. T. Zinner, J. Fortágh, and D. Petrosyan, "Optimal collection of radiation emitted by a trapped atomic ensemble". EPJ Quantum Technol. **8**, 11 (2021).
- Á. Kurkó, P. Domokos, D. Petrosyan, and A. Vukics, "Collection efficiency of optical photons generated from microwave excitations of a Bose-Einstein condensate". Phys. Rev. A **105**, 053708 (2022).
- Á. Kurkó, A. Vukics, and N. Német, "Is the photon-blockade breakdown a quantum effect? A neoclassical story". arXiv:2310.08388(2023).
- T. W. Clark, A. Dombi, F. I. B. Williams, Á. Kurkó, J. Fortágh, D. Nagy, A. Vukics, and P. Domokos, "Time-resolved observation of a dynamical phase transition with atoms in a cavity". Phys. Rev. A **105**, 063712 (2022).

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