An index to quantify an individual's scientific research output that takes into account the effect of multiple coauthorship

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I propose the index $\hbar$ ("hbar"), defined as the number of papers of an individual that have citation count larger than or equal to the $\hbar$ of all coauthors of each paper, as a useful index to characterize the scientific output of a researcher that takes into account the effect of multiple coauthorship. The bar is higher for $\hbar$.

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I. INTRODUCTION AND DEFINITION

The h-index (number of papers of an individual with citation count $\geq h$) has gained considerable acceptance as a measure of individual research achievement that is advantageous compared to other bibliometric indicators such as total number of citations or number of papers published in journals of high impact factor[1, 2, 3]. Various modifications of the h-index have been proposed to take into account some of its perceived shortcomings[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], however there is no general consensus so far that any other single number bibliometric indicator is clearly preferable to the h-index.

The h-index can be defined as follows: *A scientist has index h if h of his/her papers belong to his/her h-core. A paper belongs to the h-core of a scientist if it has $\geq h$ citations.* The h-core set is not necessarily unique because there may be more than one paper with citation count=h, but the h-index is.

In this paper I would like to define a new bibliometric indicator, $\hbar$, as follows:

A scientist has index $\hbar$ if $\hbar$ of his/her papers belong to his/her $\hbar$ core. A paper belongs to the $\hbar$ core of a scientist if it has $\geq \hbar$ citations and in addition belongs to the $\hbar$-core of each of the coauthors of the paper.

The index $\hbar$ thus defined uniquely characterizes a scientist, as the $h-$index also does, and satisfies $\hbar \leq h$. For a scientist with only single-author papers, $\hbar = h$, but $\hbar = h$ does not imply that the scientist only writes single-author papers, nor that only single-author papers form the h-core of this scientist. Furthermore, the $\hbar$-index (unlike the h-index) uniquely characterizes a *paper* as belonging or not belonging to the $\hbar$-core of its authors. (Instead, with the original h-index a multiple-author paper in general will belong to the h-core of some of its coauthors and not belong to the h-core of the remaining coauthors). Thus, the scientific literature becomes divided into two non-overlapping sets, the $\hbar$-contributing papers and the non-$\hbar$-contributing papers, and any given paper in the literature at a given time belongs to either set, never to both. As a function of time, a paper could migrate back and forth between both sets, but the vast majority of papers will either never belong to the $\hbar$ set or remain in it once they enter it. (For the nit-picking readers, "belonging" above should be replaced by "qualifying to belong", since neither the h- nor the $\hbar$-cores are unique[22]).

The $\hbar$-index just defined is somewhat difficult to understand and in addition extremely difficult to calculate. Therefore, I will define a "non-self-consistent" $\hbar$ by substituting $\hbar$ by h in the last part of the definition, namely:

A scientist has index $\hbar$ if $\hbar$ of his/her papers belong to his/her $\hbar$ core. A paper belongs to the $\hbar$ core of a scientist if it has $\geq \hbar$ citations and in addition belongs to the h-core of each of the coauthors of the paper.

In practice, the non-self-consistent $\hbar$ will be almost identical to the self-consistent one (first definition), occasionally slightly smaller. It loses some of the nice features of the first definition, in particular now a paper may belong to the $\hbar$-core of one of the coauthors and not to that of another one, although this will happen very infrequently. However with this modified definition it becomes a practical bibliometric indicator that can be calculated by hand. Thus I will only deal with the non-selfconsistent $\hbar$ in the remainder of this paper. I would like to propose $\hbar$ as a useful indicator to discriminate between scientists with different coauthorships patterns.

II. MOTIVATION

Perhaps the most important shortcoming of the h-index is that it does not take into account in any way the number of coauthors of each paper[18, 19, 20, 21, 23]. Thus, an author that publishes alone does not get any extra credit compared to one that routinely publishes with a large number of coauthors, even though the time and effort invested per paper by the single author or by each of the coauthors in a small collaboration is presumably larger than the corresponding one for a member of a larger collaboration. This can lead to serious distortions in comparing individuals with very different coauthorship patterns, and gives an incentive to authors to work in large groups when it is not necessarily desirable. For example, consider a group of several researchers that decides to put all of their names as authors in any paper they write, independently of how small or nonexistent the contribution of each of them was. The individual h-index of each of these researchers will be higher than it would have been in the absence of such decision. Of course such an extreme procedure would violate generally accepted standards of scientific integrity. Nevertheless, it is sometimes a grey area whether or not a minor contributor should be included as author of a paper; with the h-index and other current bibiometric indicators there is no penalty to add authors to a paper and as a consequence there can be an incentive to do so, due to implicit or explicit quid pro quo expectations.

Thus, a bibliometric indicator that discourages honorary authorship and gives extra credit to authors that publish alone or in small collaborations, and/or subtracts credit from coauthors in larger collaborations, would be desirable. On the other hand, the indicator should not *discourage* collaborations, that are essential for the progress of science. This is the case for example in the modification of the h-index proposed by Schreiber[18] to take into account multiple coauthorship by counting the papers fractionally according to the inverse of the number of coauthors. Similarly, Batista et al[19] proposed to divide the h index of an individual by the mean number of authors of the papers in the h-core, Egghe[20] proposed to count either citations or ranking of papers in a fractional way to take into account the number of coauthors, and Chai et al[21] discussed a scheme to allocate partial credit to each coauthor of a paper.

Such modifications of the h-index unduly discourage collaborations in this author's opinion, in addition to not being necessarily accurate or fair. In particular, there are often large differences in the individual contributions of coauthors to a joint paper, so there is no good reason to divide the credit equally among coauthors[24, 25]. It will often be the case that in a paper with a large number of citations the senior author played a crucial role, and it would be inappropriate to reduce his/her credit by an amount that depends on how many junior collaborators were involved in the project: the paper should fully contribute to at the very least that member of the collaboration that conceived and led the project. One could consider an indicator that gives large credit to the first author of the paper, who often is the main contributor, however different disciplines have different conventions on ordering of author names, in some scientific subfields (e.g. high energy physics) being almost always alphabetical.

One may be tempted to consider an indicator that gives credit only to the leading author in a collaboration (leading author meaning the leader of the collaboration, not the first author), often (not always) a senior member of the collaboration. However, it is not infrequently the case that there is more than one leading author of a collaboration. It is also often the case that very junior collaborators play a key role, in some cases they are listed as first authors and in other cases they are not. In a paper that becomes so successful that it eventually becomes part of the h-core of even the most senior member of the collaboration it would certainly be extremely unfair to deny all credit to junior collaborators that played a key role. Furthermore, such an indicator would completely discourage junior scientists from collaborating with senior scientists, with a detrimental effect on the progress of science.

On the other hand, in comparing mid-career researchers with comparable citation numbers, it may be the case that one of them achieved most of his/her citations in papers where he/she was the leading author collaborating with junior coauthors (students and postdocs), and the other one achieved most of his/her citations as a minor contributor in collaborations with more senior authors. For most evaluation purposes (eg awarding grant support or career advancement) the first researcher should be favored. This will not happen if the $h-$index is used as an indicator nor with the proposed modifications of the h-index that exist in the literature.

A useful bibliometric indicator should (i) Reflect elements of reality that are useful for evaluation and meaningful in a statistical sense (there are always exceptions to any criterion) and ideally have predictive power[26], (ii) not lead to undesirable incentives that are detrimental to the progress of science, (iii) not be too sensitive to small variations in citation records that could be due to random events, and (iv) last but not least be not too difficult to obtain from existing databases. I argue that $\hbar$ is a good candidate to meet these criteria and is superior to the h-index in that it takes into account the effect of multiple coauthorship. $\hbar$ may be used alone or in combination with the h-index.

III. CALCULATION OF $\hbar$

To understand the meaning of $\hbar$, let us consider a hypothetical example. A researcher (A) has an h-index of 20, i.e. 20 papers with 20 or more citations each, papers number 1 through 20 in order of decreasing citations. Paper 20 has 21 citations and is single author, paper 19 has 25 citations and is authored by A and a junior collaborator who himself has an h-index of 5. These papers are kept in the $\hbar$ count. Papers 18, 17 and 16 have 28, 30 and 35 citations and are coauthored with senior scientists B, C and D respectively who have h-indices 45, 55 and 40, as well as possibly other less senior (meaning of smaller h-index) coauthors. Paper 15 has 43 citations and is also coauthored with scientist D .

The $\hbar$-index will eliminate papers 18, 17 and 16 from the h-core of A because they are coauthored with senior authors whose h-index is higher than the citation counts of those papers (and of course higher than the h-index of A) . It will keep paper 15 because it has 43 citations, thus contributing also to the h-core of the senior coauthor D (that has h-index 40).

Assume for the sake of simplicity that all the other papers in the h-core of scientist A (i.e. papers 1 through 15) are either single author or coauthored with only junior collaborators (defined as collaborators with lower or equal h-index than that of A), so they all contribute fully to A's $\hbar$ count. The original h-index of 20 is now reduced by the 3 papers eliminated (18, 17, 16) to $\hbar_{first-iteration} = 17$.

This is not the end yet, because there may be other papers with citation counts between 17 and 20 (not part of the original h-core). Assume paper 21 has 19 citations and is single author, paper 22 has 18 citations and is coauthored with B, and paper 23 has 17 citations. Paper 21 should be added to the $\hbar$ count bringing it to 18, paper 22 should not be added because it has fewer citations (18) than the h-index of coauthor B (45), and paper 23 with 17 citations is lower than $\hbar = 18$ so it does not contribute, as do not contribute the subsequent papers number 24, 25, 26, ... that have citation count lower or equal to 17 by definition. Thus, for this scientist A, $h = 20$, $\hbar = 18$.

In summary: in the first iteration some papers in the h-core are eliminated yielding $\hbar_{first-iteration} \leq h$. If $\hbar_{first-iteration} < h$ a second and final iteration is needed to possibly add some papers not originally in the h-core to yield $\hbar$, with $\hbar_{first-iteration} \leq \hbar \leq h$. All $\hbar$ papers of scientist A have citation count $\geq \hbar$, and all of them contribute to the h-core of all the coauthors of each paper except possibly to the h-core of A him/herself (paper 21 in the above example). All the other papers of scientist A either have $\leq \hbar$ citations or they have $> \hbar$ citations but don't contribute to the h-core of one of the coauthors of the paper.

The calculation of $\hbar$ is surprisingly simple to perform using e.g. the ISI database. Let us go through the steps. Go to "General search", enter the name of the author. Sort the papers by "Times Cited", accordingly paper numbers $n_p = 1, 2, 3...$ (leftmost numbers on the screen) have decreasing number of citations n_{cit} for increasing n_p, and find the paper number h (i.e. the highest number paper n_p that has $n_{cit} \geq h$). Click on the title of the paper, leading to a page with the full reference and all the coauthors. Click on each of the coauthors for which there is reason to suspect that his/her h-index may be higher than n_{cit}. Find the h-index of each such coauthor, h_c. Once (if) one is found with $h_c > n_{cit}$, this paper is eliminated from the $\hbar$ count and the remaining coauthors of this paper don't have to be checked. If none of the coauthors has $h_c > n_{cit}$, the paper remains in the $\hbar$-count.

This procedure is repeated for each paper going down in the paper number of the author (corresponding to increasingly cited papers). Once a paper is reached where its citations are clearly higher than that of any coauthor the procedure is stopped. For example, in physics we know that the highest h-index is about 115 at present, so papers with $n_{cit} > 115$ certainly don't have to be checked for potential elimination.

When this procedure is completed one has identified a certain number of papers (n_d) that are deleted from the h-count. Now one goes back to paper number h, and renames that paper as being number $n_p = h - n_d$, having h or more citations. There will in general now be other papers with number $n_p > h - n_d$ that have number of citations $n_{cit} \geq n_p$, which should be included in the $\hbar$ count *provided* they are not eliminated because of having highly cited coauthors. Thus one continues in order of increasing n_p to look for the additional papers to be included with $n_{cit} \geq n_p$ until the crossing point is reached as in the usual h-index calculation. Each time a paper is eliminated due to highly cited coauthors, the numbering n_p of the subsequent papers (to be compared with n_{cit}) is reduced by 1.

Another path one can follow is to first click on the "Analyze" link to rank the records by the Author field. This will yield a list of all the coauthors of this author in order of decreasing number of papers coauthored. By looking up the h-index of the coauthors at the top of this list one can simplify the procedure described above by knowing in advance which of the frequent coauthors are high h-index coauthors that should be watched in considering whether or not a paper is eliminated from the h-count in computing $\hbar$.

IV. WHY IS $\hbar$ USEFUL?

It is clear that computing the $\hbar$ index is considerably more time consuming than computing the h-index of a researcher. But it is clearly straightforward and doable in most cases in a few minutes. In particular, it is easy for a researcher to compute his/her own $\hbar$ index, since presumably he/she has already a pretty good idea of what the h-index of most of his/her collaborators are. It should also not be too hard to compute the $\hbar$-index of some of our competitors, that may have an h-index comparable to our own but somehow we have the feeling that they are not as good as we are. Will the $\hbar$-index reflect this?

I argue that the $\hbar$-index further contributes to the 'democratization' of the evaluation and comparison of scientist's achievements, just as the h-index does compared for example to the impact factor[3], and that it is likely to give a more accurate prediction of future achievement[26]. Consider two junior researchers, one of which (A) had as thesis advisor a senior, prolific and prominent scientist running a large research operation, and the other (B) had as thesis advisor a young scientist with a small research operation. It is likely that researcher A has benefitted in getting many citations due to his/her collaboration with the more prominent scientist and his/her large group of coworkers. In addition, the letters of recommendation written by the prominent advisor of A are likely to have more weight with review committees than those written by the more junior advisor of B. If researcher A has a higher h-index than B, he/she may well be the better scientist. But assume A and B have comparable h-indices, then it is likely that scientist B has a higher $\hbar$-index, *and* I argue that it is likely that scientist B has a more promising scientific future since the larger $\hbar$ index accurately reflects his/her own higher individual abilities, not having benefitted as much as scientist A from external circumstances.

It is certainly the case that the $\hbar$-index is particularly unkind to junior researchers. If the junior researcher has as coauthor a very senior researcher that paper will not contribute to the $\hbar$ count for many years even if it is an outstanding paper and even if the junior researcher's contribution to the paper was seminal. But, if the paper is sufficiently good it will eventually contribute to the $\hbar$ count of both authors. Meanwhile, $\hbar$ should give junior researchers extra incentive to devote efforts to their independent work, or to work with other even more junior or at most contemporary collaborators, which should have a positive effect in advancing their career as well as in advancing new science. Nevertheless, in considering the bibliometric evidence for junior researchers it is especially important to use the $\hbar$ index *together* with the h-index as well as other bibliometric as well as non-bibliometric criteria. If one insists on a single bibliometric parameter perhaps an average of the h and $\hbar$ indices would be appropriate.

Note also that in collaborations where the authors have similar h-index, the collaboration will not result in a much reduced $\hbar$-index for any of the collaborators. For example, if the h-index of coauthor B (h_B) is slightly higher than that of author A (h_A), only the papers of A and B whose citation number fall in the narrow interval $[h_A, h_B]$ can be eliminated from the h-core of A in computing his/her $\hbar$. Thus, the $\hbar$-index does not penalize collaboration between scientists of similar seniority, unlike the other proposed modifications of the h-index to take into account multiple coauthorship[18, 19, 20, 21]. Such collaborations are often very fruitful.

On the other hand, $\hbar$ clearly discourages honorary authorship for more senior/prominent researchers, and it also discourages collaborations for the sole purpose of increasing the coauthors' h- or $\hbar$-indices even among collaborators with similar h-indices. Unlike the h-index, the $\hbar$-index of a researcher can decrease with time. This will happen if a coauthor's h increases sufficiently so that a coauthored paper can get eliminated from the $\hbar$ count. Even though for coauthors with similar h-indices their $\hbar$ indices are not reduced, $\hbar$ could be reduced for one of them in the future if the coauthor's h-index increases substantially.

The $\hbar$ index should be more accurate than the h-index in reflecting the individual contributions of a researcher where he/she played a leading role, as well as those where he/she is likely to have played an important enough role to have propelled the paper to the h-core of even its more senior coauthors. As defined, the $\hbar$-index is probably almost useless for scientists at the stage of post-doc and of very limited use at the beginning assistant professor stage. However I believe that it can start playing a significant role at the time of evaluation for promotion to tenure, where individual independent research contribution should be evaluated, as well as further along in the scientist's career. Furthermore I argue that $\hbar$ should play a useful role together with the ordinary h-index and other criteria, for decisions on allocation of research resources by granting agencies throughout a scientist's career except perhaps at the very beginning.

TABLE I: Publication data for a recently tenured physicist at UCSD: $h = 17$, $\hbar = 12$. h_{max} denotes the h-index of the highest-h collaborator of the paper (senior and single mean the physicist in question was senior author or single author of that paper). $n_p^{\hbar}$ counts the papers in the $\hbar-$core. The last column gives the publication year of the paper.

n_p	n_{cit}	h_{max}	coauthors	$n_p^{\hbar}$	n_{cit}	paper year
1	229	41	3	1	229	1996
2	186	41	2	2	186	1996
3	70	80	3	-	70	2001
4	67	41	2	3	67	1995
5	63	senior	2	4	63	1997
6	43	41	4	5	63	1997
7	39	41	3	-	39	2004
8	28	60	3	-	28	2000
9	26	41	2	-	26	2007
10	25	107	3	-	25	2006
11	24	43	5	-	24	2008
12	23	senior	4	6	23	2008
13	22	41	2	-	22	1994
14	20	36	3	-	20	2008
15	19	50	6	-	19	2000
16	18	single	2	7	18	2004
17	17	single	1	8	17	2002
18	15	senior	2	9	15	2005
19	15	single	1	10	15	2005
20	14	33	7	-	14	2006
21	13	single	1	11	13	2005
22	12	senior	2	12	12	2005
23	11	28	3	-	11	2006
24	10	41	2	-	10	1998
25	9	single	1	13	9	1998

V. EXAMPLES

Let us now consider some examples. As a first illustrative example I give in table I the publication record (25 highest cited papers) of a recently tenured physicist at UCSD. His scientific age is $n = 15$, total number of papers 51, total citations 1074, $h = 17$ and $\hbar = 12$. h_{max} in each row gives the highest h-index among the coauthors of that paper, "senior" and "single" mean that the physicist in question is either the senior member of the collaboration or the single author, hence equivalent entries would read $h_{max} = 17$. It can be seen that the 12 papers contributing to $\hbar$ are a healthy mix of very highly cited papers with senior coauthors ($n_p = 1, 2, 4$ and 6), and papers where this scientist is senior or sole member of the collaboration ($n_p = 5, 12, 16, 17, 18, 19, 21, 22$). Highly cited paper number 3 (70 citations) is eliminated from the $\hbar$-count due to the very high h of a collaborator, $h = 80$, as are several others ($n_p = 7, 8, 9, 10, 11, 13, 14, 15$) . Several of these coauthored papers that are presently part of the $h-$core but not of the $\hbar$-core will eventually become part of the $\hbar$-core once/if their number of citations exceeds h_{max} for that row (which will of course also increase with time in general). Four papers that were not in the h-core ($n_p = 18, 19, 21, 22$) get added to the $\hbar$-core, due to the fact that only 8 out of 17 papers in the h-core survived the extra requirement needed to belong to the $\hbar$-core. These are all fairly recent senior- or single-author papers, indicating that the researcher is becoming increasingly independent.

It would be unfair and counterproductive in this author's opinion to make this researcher "pay" in his bibiliometric index because of having collaborated with his graduate students and postdocs in his papers $n_p = 5, 12, 18, 22$ instead of being sole author, as would be the result of applying any of the fractional credit schemes proposed in the literature to account for multiple coauthorship[18, 19, 20, 21].

It is possible in this example that the self-consistent $\hbar$ is slightly higher than the non-self-consistent one: the easiest way for this to happen would be if $h_{max} = 41$ in paper $n_p = 7$ would correspond to an $\hbar$ for that scientist of 39 or lower, in which case that paper would belong to the self-consistent $\hbar$ core of all the coauthors of the paper including the scientist under consideration.

D.J. Scalapino is a highly productive leading senior researcher in condensed matter theory ($h = 81$) that has worked with a large number of collaborators. As a second set of examples, Table II shows the h and $\hbar$ indices for Scalapino and 25 of his collaborators (identified by their initials), with the last column giving the scientific age of the scientist (time since first published paper). Scalapino himself has $\hbar = h$, being the most senior researcher in all his highly cited coauthored papers. The h indices of Scalapino's collaborators in table I range from 59 down to 7, the $\hbar$ indices from 58 to 1, and $\Delta h = h - \hbar$ for a given researcher ranges from 10 to 1. It should be pointed out that the reduction from h to $\hbar$ in the various cases occurred not only because of collaboration with Scalapino but also with several other senior researchers in condensed matter theory. Table II is arranged in order of decreasing h index.

The following features are interesting to note:

(i) There is not a very clear correlation between Δh and h. For example, SRW with $h = 47$ has $\Delta h = 10$, EJ with $h = 14$ has $\Delta h = 4$. However, it is true that very junior researchers have a very large Δh compared to their h.

(ii) There is also not a clear correlation between Δh and n, scientific age. ED with $n = 26$ has $\Delta h = 1$, RLS with $n = 42$ has $\Delta h = 5$. Thus it is certainly not generally true that very senior researchers will always have converging h and $\hbar$.

It can be seen from the $\hbar$ column that in several cases the h and $\hbar$ ranking orders are reversed, in particular:

(iii) SRW has $h = 47$, larger than RLS's $h = 43$, yet RLS's $\hbar = 38$ is larger than SRW's $\hbar = 37$.

TABLE II: h and $\hbar$ indices for D.J. Scalapino (condensed matter theorist, $h = 81$) and 25 of his collaborators. $\Delta h \equiv h - \hbar$, and n is the scientific age $\equiv$ number of years since first published paper.

Researcher	h	$\hbar$	Δh	n(years)
DJS	81	81	0	45
ED	59	58	1	26
SRW	47	37	10	23
AM	43	35	8	25
RLS	43	38	5	42
DP	35	27	8	23
RTS	34	29	5	24
MJ	34	33	1	23
WH	34	30	4	38
AVB	30	26	4	25
PJH	27	25	2	23
AWS	27	23	4	18
JEG	25	23	2	38
JKF	24	21	3	21
NEB	23	17	6	24
TPD	23	20	3	18
RMN	21	16	5	18
TD	21	11	10	17
MJM	21	19	2	23
PM	20	13	7	19
NB	20	12	8	20
EJ	18	14	4	16
FFA	16	10	6	19
LC	14	8	6	12
TAM	11	5	6	7
SG	7	1	6	7

(iv) DP's $h = 35$ is larger than that RTS's h=34, MJ's h=34 and WH's h=34, yet these researcher's $\hbar$ of 29, 33 and 30 are all larger than DP's $\hbar = 27$.

(v) MJM's h=21 is smaller than NEB's h=23, yet MJM's $\hbar = 19$ is larger than NEB's $\hbar = 17$.

(vi) EJ's h=18 is smaller than NB's h=20, PM's h=20 and TD's h=21, yet EJ's $\hbar = 14$ is larger than NB's $\hbar = 12$, PM's $\hbar = 13$ and TD's $\hbar = 11$

Furthermore, in several cases where the h-indices are identical for two researchers the $\hbar$ indices are quite different, for example:

(vii) AM and RLS have both h=43, but RLS's $\hbar = 38$, AM's $\hbar = 35$.

(viii) MJ, RLS and WH all have $h = 34$, yet their $\hbar$'s are 33, 30 and 29 respectively.

(ix) PJH and AWS have both $h = 27$, yet their $\hbar$'s are 25 and 23 respectively.

(x) TPD and NEB have both $h = 23$, yet their $\hbar$'s are 20 and 17 respectively.

(xi) MJM, RMN and TD have all $h = 21$, yet their $\hbar$'s are 19, 16 and 11 respectively.

Note that MJ, MJM and EJ have particularly small values of Δh compared to their similar-age peers. When reordering the entries in Table II according to decreasing $\hbar$ rather than decreasing h, they move up 3 ranks. I suggest that their higher $\hbar$ values reflects higher individual accomplishment and promise of future accomplishment than is reflected in their h-index, i.e. compared to peers with similar h-index and larger Δh. The future will tell.

As a final example I discuss the case of high energy experimental physicists. For these scientists the h-index is essentially meaningless because they usually work in collaborations with hundreds of physicists, and everybody's name is listed in the author's list. The h-index of mid-career and senior successful high energy experimentalists is typically in the 40's and higher. However, it is reasonable to expect that their $\hbar$ index will be much lower. To test this expectation I attempted to calculate the $\hbar$ index of a senior high energy experimentalist at my institution, that has $n = 34$ (first paper published in 1975), 334 publications, and h=44. Going down from $n_p = 44$ progressively to lower n_p values the citation count increases very slowly and papers are eliminated because of coauthors with higher h, eg. G. Coignet (h=49), D. MacFarlane (h=54), J.D. Burger (h=64). Paper number 18 in the publication list has 63 citations (in going from paper 44 to paper 17, citation count increases only from 44 to 64!), so paper 17 gets eliminated from the $\hbar$-count because Burger is a coauthor. Therefore, the first iteration yields $\hbar = 17$ *or lower*. Assuming no paper below $n_p = 17$ gets eliminated (I did not check that fully), it is now necessary to consider the papers with $n_p = 45$ and higher to see if there are papers with $n_{cit} > 17$ that don't get eliminated by having high h coauthors. One finds that papers 45 to 166 have n_{cit} ranging from 43 down to 18, paper 167 has $n_{cit} = 17$, and all these papers are eliminated by having coauthors with h higher than n_{cit}, not surprisingly (since almost every paper has hundreds of coauthors). The conclusion is then that for this researcher $h = 44$ and $\hbar = 17$ (or even lower in case there are higher h coauthors in papers 1 to 17 that I missed), i.e. $\Delta h = 27$. It is reasonable to expect that such a large reduction from h to $\hbar$ will be the norm for high energy experimental physicists, as well as in other fields where collaborations typically involve a very large number of coauthors.

VI. CLOSING ARGUMENTS

$\hbar$ gives full credit to the senior coauthor of a paper (senior meaning the coauthor with highest h-index), where by 'full credit' I mean the same credit that h would give. $\hbar$ may or may not give any credit to a junior coauthor of a paper who does get credit for that paper in his/her h-index. Thus, $\hbar$ will give deserved extra credit to those young and mid-career scientists that lead vigorous independent research programs compared to those that don't, encourage them to take on younger students and postdocs without any penalty, and discourage them from instead spending a lot of their effort in collaborations in research projects led by more senior scientists, as well as discourage them from including the name of senior/prominent scientists in their papers as coauthors (as opposed to e.g. in an acknowledgement for a minor contribution) for purely political reasons. The author believes that these incentives are fair and beneficial to the progress of science. It is likely that scientists with higher $\hbar$ than colleagues with the same h will make better use of research funds allocated to them by granting agencies. Making such authors "pay" in their bibliometric indices[18, 19, 20, 21] for working with students and postdocs is neither fair nor beneficial to anybody nor does it yield any useful information about these scientists.

It may be reasonably argued that for junior scientists, papers with senior coauthors should be taken into account at least fractionally while they are part of the h-core but not of the $\hbar$-core rather than not counted at all. That is easily achieved by using as indicator some weighted average (for example the arithmetic average with equal weights) of h and $\hbar$ as defined here. The weights in such an average may even be taken to be time-dependent, with the relative $\hbar$-weight starting from zero and increasing with time. It may be argued that whether or not a paper in the h-core makes it to the $\hbar$-core should depend on the publication date of the paper relative to the present. However such algorithms would become very complicated.

To the extent that grant awarding agencies and faculty recruitment and advancement committees consider the information provided by the h-index in awarding grant support and evaluating job candidates and career advancement, I believe it is imperative that they consider the information provided by $\hbar$ also. It would be both unfair and not conducive to optimal results (i.e. advancement of the best science) to favor a candidate that achieved a high h index predominantly by joining collaborations with prominent senior colleagues over another one that achieved perhaps a somewhat smaller h but a substantially higher $\hbar$ through mostly leading his/her own independent research effort with junior colleagues.

$\hbar$ is very harsh on the very young scientist, and as mentioned earlier it should only be used if at all in such cases in combination with the h-index and other indicators, as well as with other non-bibliometric criteria. One may fear that $\hbar$ will unduly discourage young scientists from collaborating with senior and/or prominent scientists. However, there are plenty of other incentives for young scientists to collaborate with senior/prominent scientists, namely the availability of resources, the benefits of learning from top scientists, and the letters of recommendation to be obtained from these influential members of the scientific establishment, to name just a few. In the light of $\hbar$, the young scientist collaborating with senior scientists should regard his/her effort as a long term investment, that may pay off (in increasing his/her $\hbar$) eventually, in addition to providing the shorter term benefits just mentioned.

There will undoubtedly be cases where $\hbar$ will do injustice. For example, a young theorist that interacts closely with a senior experimentalist and makes suggestions for new data taking may write theoretical papers explaining the experimental data where the experimentalist is a coauthor, and the paper will not contribute to the theorist's $\hbar$-count for many years; another similar theorist interacting less closely with the experimentalist may just use the available experimental data and thank the experimentalist in an acknowledgement, thus having the paper count much earlier to his/her $\hbar$-index. Such situations are obviously beyond the grasp of any necessarily coarse-grained bibiliometric indicator, and highlight the need for evaluators to consider individual circumstances in each case in addition to the indicators.

$\hbar$ gives full credit to all members of a collaboration once the paper garners enough citations to contribute to everybody's h-core. It may be argued that even in that case the contribution of some members of the collaboration might have been so insignificant that the credit is undeserved. This may be true in some cases, but more often than not in very successful papers all participants are likely to have played important roles in making the paper successful, and if not, well - so be it.

$\hbar$ is not very friendly to high energy experimentalists nor presumably to researchers in other fields where collaboration among a very large number of scientists is the norm. I believe this means that a low $\hbar$ value for a high energy experimentalist (low compared to a non-high-energy experimentalist) should not be interpreted as shedding negative light on the scientist, and a high h value for a high energy experimentalist (high compared to a non-high-energy experimentalist) should not be interpreted as shedding positive light on the scientist. In a nutshell, neither h nor $\hbar$ indices are very useful for high energy experimentalists except perhaps for comparison with other high energy experimentalists, and the same should be true for other regular large-group collaborators.

It should be interesting to explore the time evolution of h and $\hbar$ for individual scientists. At the beginning of a scientist's career there will usually be a large difference ($\hbar$ much smaller than h), and as the career advances the "hbar-gap" should gradually close ($\Delta h \to 0$) as $\hbar$ and h indices converge, the more so the more independent and successful the scientist is. As an extreme example, for Ed Witten, who has the highest h-index among physicists, it is clear by definition that $\hbar = h$ (unless he collaborated with e.g. a biologist with much higher h-index). It is likely that almost all leading senior scientists have $\Delta h = 0$ or very small. Moreover, in comparing scientists across disciplines[27] that have different citation patterns and different typical values of h, scientists with $\Delta h \sim 0$ will typically be the leaders in their discipline.

Note that the $\hbar$-index will "weed out" from the individual h-index predominantly those papers that have citation count close to h, and will always keep those papers with very large number of citations independent of their authorship. Therefore it will give relatively higher weight to very highly cited papers relative to lower cited papers, both of which contribute equally to the h-index. This is similar to what is aimed at in some of the proposed modifications to the h-index, e.g. Egghe's g-index[4], that weighs highly cited papers more.

In fact, the concept of the $\hbar$ index can also be applied in an identical way to any of the other indices that have been proposed as alternatives to the $h-$index. For example, Egghe's g-index defines the g-core as the highest number of papers g that received *on average* g or more citations. The "gbar" index would add the requirement that each paper should belong to the "gbar" core of all its coauthors (or to the g-core for the non-self-consistent version).

As mentioned earlier, the $\hbar$ index can decrease with time. A young scientist collaborating with similar-aged peers may see his/her $\hbar$-index gradually *decrease* in later years if his/her former collaborators vastly outperform him/her in scientific achievement in later years. Unlike with h, "we must keep running to stay in the same spot".

The calculation of (the non-self-consistent) $\hbar$ using information provided in the ISI and Scopus databases is feasible with moderate effort (typically a few minutes compared to a few seconds for the calculation of h). It would be facilitated if ISI and Scopus would provide the h-indices with fewer needed clicks, e.g., if in clicking on the paper title the list of authors appeared together with their respective h-indices. That should not be very computer-time consuming. Currently in ISI one needs two further clicks to reach the h-index of each coauthor (clicking on the coauthor's name, and then on the "Create Citation Report" link). The latter can be quite time-consuming because it calculates many other things in addition to the h-index, and in fact it is often much faster to calculate the h-index of each coauthor by hand, as originally described[23].

The self-consistent $\hbar$ is an appealing theoretical construct but has no practical value since it appears extremely difficult to compute, involving an iterative process including an ever-growing number of scientists. It is however likely to be identical or very close to the non-self-consistent $\hbar$ considered here in almost all cases, always an upper bound.

In the paper where I introduced the h-index[23], I suggested that in physics reasonable values of h (with large error bars) might be $h \sim 12$ for advancement to tenure (associate professor), $h \sim 18$ for advancement to full professor, $h \sim 15-20$ for fellowship in the American Physical Society and $h \sim 45$ for membership in the US National Academy of Sciences (NAS). In hindsight, the estimate for fellowship in the American Physical Society was much too low, $h \sim 20 - 25$ being more typical values[26]. One may wonder about similar estimates for $\hbar$. For membership in NAS it should not be very different, since it is usually late in a scientist's career. If the $\hbar$ of a scientist proposed for membership of the NAS is much smaller than his/her h, this should raise red flags about his/her independence. For fellowship in the APS $\hbar$ should perhaps be $\sim 17 - 22$ and for advancement to full professor perhaps $\hbar \sim 15$ or 16. For tenure, an $\hbar$ as low as 7 or 8 may be sufficient especially if h is substantially higher.

One of the attractive features of the h-index is that equally good papers (i.e. papers acquiring citations at the same rate) will give credit to a young author much faster than they do to an older author, that has a higher h bar to overcome. That attractive feature is preserved in $\hbar$ if there are no senior coauthors and lost if they are. But, on the other hand it may be argued that a young person has the luxury of many more years ahead in his/her career, and in a collaboration involving junior and senior coauthors a good paper may contribute to the $\hbar$-count of the young coauthor when it is already too late for the senior coauthor to benefit from it in either h or $\hbar$ (i.e. after retirement or worse). Young scientists with an independent bent will probably love $\hbar$, established scientists that benefit from collaboration with young coauthors even when their (the established scientist's) participation is minor will probably hate it, since to the extent it is used as an evaluation tool it will make junior authors more reluctant to include senior coauthors in their papers. No bibliometric index will ever make everybody happy. In the end, whether h and $\hbar$ have a beneficial or detrimental effect on the scientific enterprise will depend on how they are used, and hopefully the scientific community and the scientific administration community will converge to the right uses to provide optimal incentives for the advancement of science.

The bar is higher for $\hbar$. For a paper that counts for your h to be in your $\hbar$ count, either you have to be sole author or the senior author of the paper, or else you as well as your other junior and senior coauthors should have contributed enough bang for the buck to make it a significant paper for each and every one of the coauthors involved. A vanishing hbar-gap in the advanced stages of a scientist's career is a hallmark of scientific leaders.

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